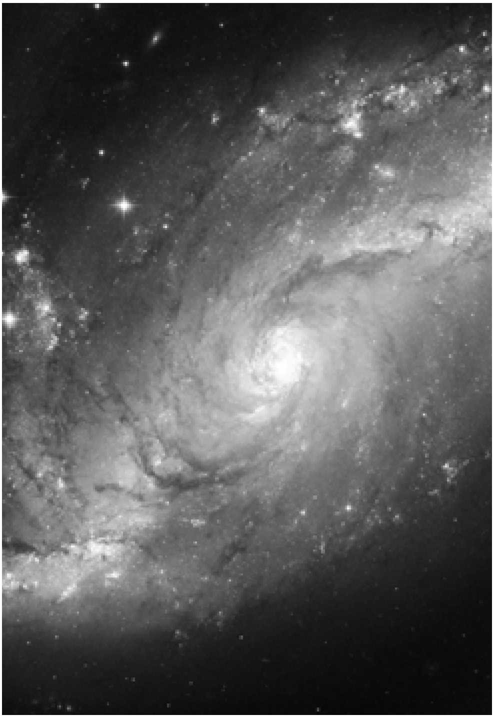
CONSTRUCTION OF A GEOMETRY IN RELATIVITY



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FOREWORD

The Usefulness of a New Geometry in Relativity

The currently used geometry is extremely complex (it involves constructing a basis of the dual of the space considered as a vector space and tensor calculus) and does not allow for the representation of space in an accelerated change of reference frame or under the influence of a gravitational field. It only allows the representation of a single point.

Hence, it is impossible to describe the universe in Relativity and, consequently, to describe the motion of an object subjected to forces (position, velocity, acceleration at each instant).

Through a very classical geometric construction, another, much simpler geometry can resolve these difficulties.

It leads to the perception of space having a dual structure, provides an explanation for the acceleration of the expansion of the universe, and offers a step towards understanding the reason behind the invariance of the speed of light: this physical phenomenon appears to be directly linked to the dual structure of space.

These results are based on a classical and rigorously reasoned mathematical approach.

CONSTRUCTION OF A GEOMETRY IN RELATIVITY

Scientific study by Daniel Senequier

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Presentation of this approach

This study lies halfway between mathematics and physics. It is not about reflecting on physics in the theory of Relativity.

This physics has been perfectly known and verified for over a century. We fully respect it. It is also not about constructing a new mathematical theory but simply using mathematical tools, which are also perfectly known, to attempt a less analytical and more geometric approach. Analytical calculus, widely used in the geometry of Relativity, often yields numerical results that are challenging to interpret geometrically.

What we aim to demonstrate is, on the one hand, that geometry in Relativity can be approached through an entirely geometric perspective, independent of physics, with a completely geometric representation that allows us to bypass tensor calculus in linearly accelerated reference frames, significantly reducing the need for analytical calculations. On the other hand, we want to show that the structure of space is more complex than the perception we have of it in the Euclidean reference frame.

This geometry is particular in that the norm defined in a Euclidean frame is unstable; it changes when there is a change of reference frame if the two reference frames are in relative motion. This leads to greater geometric difficulties when the moving reference frame is accelerated. The isomorphism that exists between space, considered as a vector space, and its dual containing the mechanical laws considered as linear forms is broken. To restore this isomorphism, the dual must be transformed in the same way as the vector space, which means constructing a basis for the dual and using tensor calculus. This mathematical framework is quite cumbersome.

The geometric difficulties, related to the breaking of the Euclidean norm during a change of reference frame, are linked to the well-known and verified physical phenomenon of the invariance of the speed of light. If this particular physical phenomenon did not exist, there would be no break in the Euclidean norm during a change of reference frame. The transformation laws for transitioning from a fixed reference frame to a moving reference frame would result in a simple translation (an affine transformation with a ratio of 1) and/or a rotation that only changes the axes. Mechanics would be very similar to classical mechanics, with the only difference being a translation of the time expression during a change of reference frame:

$$t' = t - \frac{vx}{c^2}$$

t' = time in the moving reference frame, t = time in the fixed reference frame fixe, v = relative velocity of the moving reference frame, x = abscissa of the point.

The phenomenon of the invariance of the speed of light is mathematically represented by the square root $\sqrt{1 - \frac{v^2}{c^2}}$ which disrupts the Euclidean norm.

In our geometric approach, we aimed to transform this analytical expression into a trigonometric one. The idea is not entirely new, as MINKOWSKI's theoretical development is based on a similar approach. However, MINKOWSKI relies on hyperbolic trigonometry.

We used circular trigonometry by introducing a change of variable $\sin\theta = \left|\frac{v}{c}\right|$ with $0 \leq \theta < \frac{\pi}{2}$.

Under these conditions, we have $\sqrt{1 - \frac{v^2}{c^2}} = \cos\theta$, which leads in imagining a particular rotation of angle θ. To achieve this, we envisioned that space, considered as a vector space, is embedded in a higher-dimensional vector space (all of this is developed in the theoretical section that follows), and we constructed a rotation within it. Space then appears as a direct sum of two vector subspaces (supplementary vector subspaces).

The first of these vector subspaces is identical to the vector space contained in the fixed reference frame (for this reason, we called it the «real» space). It corresponds to what the vector space contained in the moving reference frame would be if the phenomenon of the invariance of the speed of light did not exist. The second vector subspace is directly linked to the phenomenon of the invariance of the speed of light.

Furthermore, this vector space is derived by a linear transformation (a combination of rotation and homothety) from a part of the «real» space. We referred to it as the «supplementary space». The parameters within it appear with an opposite sign to those in the «real» space.

The representation of space thus takes on a completely geometric form, and its study relies relatively little on analytical calculations. We have demonstrated that it is possible to reason in geometry solely within the «real» space and derive a complete representation of space. To do this, we use the geometric properties of the «supplementary space», deduced from the «real» space by the linear transformation mentioned above, and then we perform vector summation.

We have shown that the same process of geometric reasoning in the «real» space, followed by an extension through a linear transformation to the «supplementary space», also applies to mechanics, even in accelerated reference frames. It is possible to construct a complete image of space in accelerated reference frames without the need to build a basis for the dual and use tensor calculus.

When the trajectory is curved, the calculations become more complex because it is necessary to continuously relate the conditions to those of an axial trajectory through the use of rotations and translations with respect to the fixed reference frame.

We have also calculated the radius of curvature of space at a given time t in reference frames with axial acceleration and defined the center of curvature.

The expression of energy (mass) is linked to the choice of reference frame. Energy is not an intrinsic parameter; its nature is geometric. We have chosen to provide a vector representation of it. This representation only changes its expression along a normalized axis. Thus, when the mass appears in vector form, we apply the same rotation to it as to the other space parameters (geometric space and time). The mass then appears invariant in the «real» space. However, after the same rotation as the other space parameters, its expression is transformed by the same homothetic transformation.

Thus, mechanics appearing in the “real” subspace would be identical to the mechanics of NEWTON if the measurement of time were universal, i.e. if the speed of light were infinite. The only difference with the mechanics of NEWTON in the “real” subspace would result in this context from the finitude of the speed of light. The not empty supplementary subspace whose existence is associated with the invariance of the speed of light, contains physical and geometrical parameters (parameters of geometrical space, of time and of energy). It thus has a physical and geometrical reality, perceived in the moving reference frame and not perceived in the fixed reference frame. It shows that the structure of the space is more complex than the perception which we have of it in the Euclidian reference frame.

This supplementary subspace moreover has properties of symmetry : the parameters appear in with a negative sign. The mass of negative value suggests a negative gravitation. It thus appears in the above mentioned homothety and applied to mechanics.

The phenomenon of invariance of the speed of light appears thus as an interaction between the light propagation (propagation of the electromagnetic waves) and a not directly perceived symmetrical space.

Théorie générale

Let us consider, temporarily, a space of dimension 3 and, temporarily also, within the framework of the simple relativity with a reference frame of LORENTZ.

We thus have two Euclidian reference frames in uniform linear relative movement. One is qualified "fixed" and the other "mobile".

We now regard the vector space described in these two reference frames as vector subspace of a vector space of higher size, vector space in-which it is imbedded.

We are going to define a particular rotation in this extended space.

Construction of the new reference frame :

The 2 reference frames are $R(O,x,y,z)$, «fix» and

$R'(O',x',y',z')$, mobile v represents the speed of translation of R' with regard to R, and $-v$ the relative speed of R with regard à R', c represents the speed of light propagation.

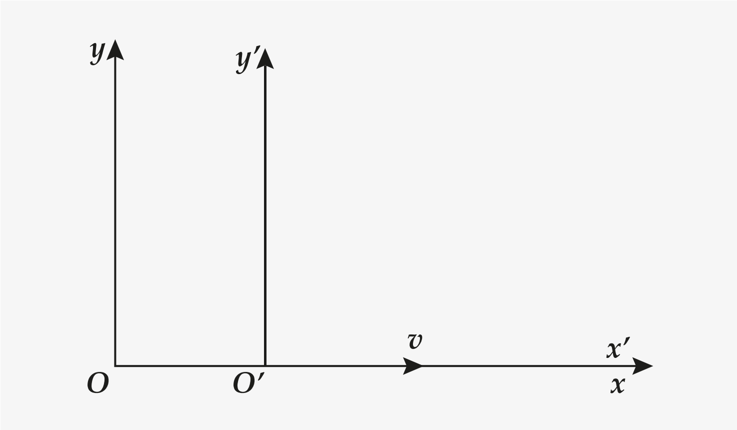

fig. 1 : Fixed and mobile reference frames

The equations of transformation of LORENTZ are written :

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

We are going to complete each of the real spaces E and E', defined by the reference frames $R(O,x,y,z)$ and $R'(O',x',y',z')$ by a supplementary space. Supplemented spaces are indicated by Ec and $E'c$. With this intention, we use the properties of vector space of Euclidean space.

We extend them to supplemented space.

$$R\ (O, x, y, z) \mapsto Rc\ (O, x, y, z, \bar{x}, \bar{y}, \bar{z})$$

$$R'\ (O', x', y', z') \mapsto R'c\ (O', x', y', z', \bar{x}', \bar{y}', \bar{z}')$$

We have the scalar product in the Euclidian space. It is a symmetric bilinear form, that can be defined in a vector space of any dimension. Its quadratic form allows to define the orthogonality, the norm and the distance :

Let us extend the scalar product to Ec and $E'c$.

Given V_1 and V_2 two unspecified vectors of Ec and $E'c$. The condition of orthogonality results in :

$$V_1 \perp V_2 \Leftrightarrow V_1 \cdot V_2 = 0$$

The norm results from the quadratic form :

$$\| V \| = \sqrt{\sum (x_i^2)}, (i = 1, 6);$$

In addition : $\| V \| = 0 \Leftrightarrow V = 0$ (V = isotropic vector)

We can define the angle of 2 nonnull vectors from the scalar product : given V_1 and V_2 two nonnull vectors, belonging both either to Ec, or to $E'c$, the angle θ between these 2 vectors is defined by :

$$\cos \theta = \frac{V1 \cdot V2}{\|V1\| \cdot \|V2\|}$$

We thus suppose the 6 axes of each reference frame $Rc\ (O, x, y, z, \bar{x}, \bar{y}, \bar{z})$ and $R'c\ (O', x', y', z', \bar{x}', \bar{y}', \bar{z}')$ orthogonal and having a norm within the meaning of the scalar product, which makes it possible to define unit vectors on each axis.

The reference frames Rc and $R'c$ thus form each one an orthonormal basis, in Ec and $E'c$ respectively.

We call Er and $E'r$ the restrictions of Ec and $E'c$ respectively in the “real” space (formerly E and E'), Ev and $E'v$, the restrictions of Ec and $E'c$ in the “supplementary” space.

Er, $E'r$, Ev and $E'v$ thus constitute vector subspaces of Ec and $E'c$.

Their sums constitute Ec et $E'c$.

$$Ec = Er + Ev \quad ; \quad E'c = E'r + E'v$$

The basis Rc, $R'c$, Rr, $R'r$, Rv, $R'v$ of Ec, $E'c$, Er, $E'r$, Ev, $E'v$, respectively constitute the basis of corresponding vector spaces and subspaces.

Rotation of axis:

We will superimpose a reference frame $R''c$ with the reference frame $R'c$, in order to define a rotation in space $E'c$. The reference frame $R''c$ is identical to the reference frame $R'c$. Its O'' origin coincides with O'.

It is provided with axis $O''x''$, $O''y''$, $O''z''$, $O''\bar{x}''$, $O''\bar{y}''$, $O''\bar{z}''$, and with the same norm as $R'c$.

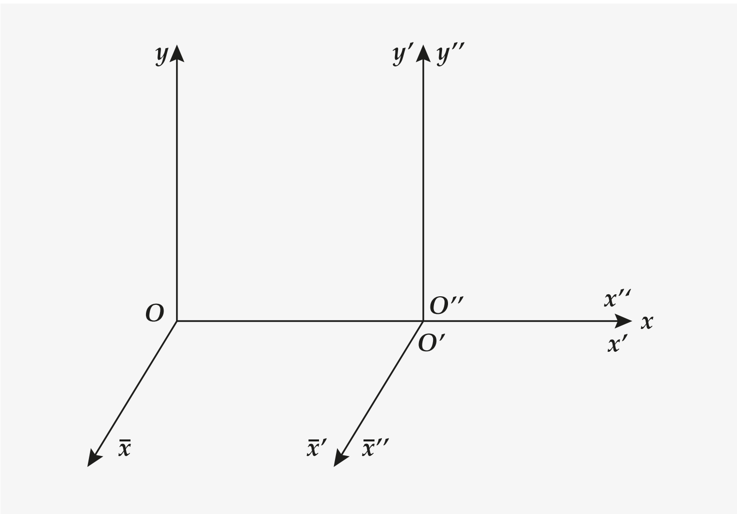

fig. 2 : Juxtaposed reference frames

The coordinates of any point P of $E'c$, are :

$(x', y', z', \bar{x}', \bar{y}', \bar{z}')$ in $R'c$.

They become $(x'', y'', z'', \bar{x}'', \bar{y}'', \bar{z}'')$ in $R''c$:

$$x' = x'' \;;\; y' = y'' \;;\; z' = z'' \;;\; \bar{x}' = \bar{x}'' \;;\; \bar{y}' = \bar{y}'' \;;\; \bar{z}' = \bar{z}''$$

We are interested in the expression $\sqrt{1 - \frac{v^2}{c^2}}$

Given $\sin\theta = \left|\frac{v}{c}\right|$; $0 \leq \theta < \frac{\pi}{2}$

We will consider the so chosen limits later on.

We have : $\sqrt{1 - \frac{v^2}{c^2}} = \cos\theta$

We make a rotation of angle θ of the reference frame $R''c$ in the plan $(O''x'', O''\bar{x}'')$. Other coordinates remaining unchanged, we can write the equations of change of reference frame in the plan $(O''x'', O''\bar{x}'')$.

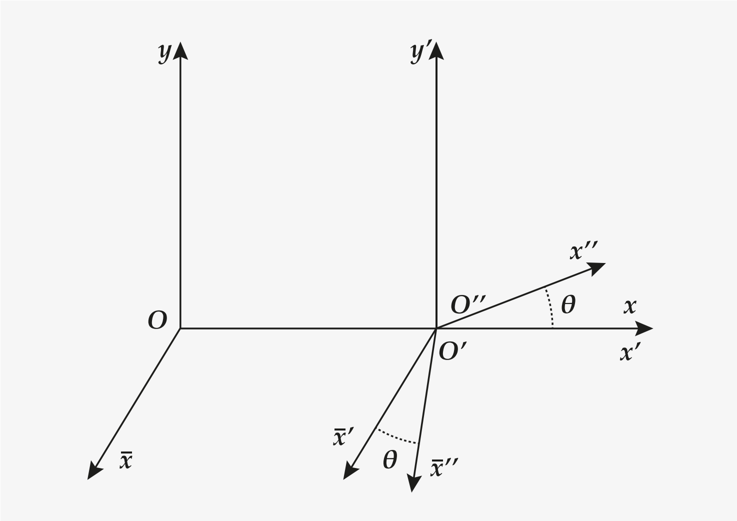

fig. 3 : Rotation of the mobile reference frame

The equations which give the rotation are written :

$$\begin{pmatrix} x'' \\ \bar{x}'' \end{pmatrix} = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} x' \\ \bar{x}' \end{pmatrix}$$

with $\bar{x}' = 0$

We have :

$$x'' = x' \cos(\theta)$$
$$\bar{x}'' = -\, x' \sin(\theta)$$

But, $x' = \dfrac{x - vt}{\sqrt{1 - \dfrac{v^2}{c^2}}}$; $\sqrt{1 - \dfrac{v^2}{c^2}} = \cos(\theta)$

$\Rightarrow$ $x' \cos(\theta) = x - vt$

$$\Rightarrow \left[\begin{array}{l} x'' = x - vt \\ \bar{x}'' = -(x - vt)\, tg(\theta) \end{array}\right.$$

The coordinates of any point P of the space in $R''c$ are :

$$x'' = x - vt$$
$$y'' = y$$
$$z'' = z$$
$$\bar{x}'' = -(x - vt)\, tg(\theta)$$
$$\bar{y}'' = 0$$
$$\bar{z}'' = 0$$

Let's revert to the domain of definition of the angle θ :

($0 \leq \theta < \dfrac{\pi}{2}$) :

The change of variable was the following :

We had put $\sin^2(\theta) = \dfrac{v^2}{c^2}$.

We have : $\sin(\theta) = \left|\dfrac{v}{c}\right|$ with $\sin(\theta) \geq 0$.

The definition of θ being more or less $2k\pi$, we will keep to it the included values between 0 and 2π ;

$\sin(\theta) \geq 0$ imposes $0 \leq \theta \leq \pi$.

The value $\theta = \dfrac{\pi}{2}$ is forbidden because $v < c$.

The domain of possible definition of θ is thus

$\left[0, \dfrac{\pi}{2}\right]$ and/or $\left[\dfrac{\pi}{2}, \pi\right]$.

Considering the necessity of a bijection between θ and v, the domain of definition of θ will belong either to the first or to the second of the above domains.

If the second domain is chosen, when
$sin(\theta) \rightarrow 0 \Rightarrow \theta \rightarrow \pi$

Thus, and for reasons of continuity,
$v \rightarrow 0 \Rightarrow \theta \rightarrow \pi$

For $v = 0$, the change of variable induces a rotation of π, c'est à dire une inversion de sens des axes de coordonnées. i.e. an inversion of sense of axis. Now, and considering the initial hypothesis, for $v = 0$, the axis of coordinates must be oriented in the same sense (reference frame of LORENTZ).

Thus, the choice of θ in the domain $\left[\frac{\pi}{2}, \pi \right]$

must be excluded.

It results :

1 - that $0 \leq \theta < \frac{\pi}{2}$

2 - that the values of the parameters in supplementary space thus are well preceded by the minus sign.

Geometrical interpretation

In $R''r$, after rotation, we have an image of the space which is identical to that of the space in Rr. It only appears a translation of the reference frame (affine transformation) identical to the one who would appear in classic (Newtonian) mechanics.

The strictly «relativist» property, the consequence of the invariance of the speed of light in the change of reference frame, is transferred in the supplementary space and projected in the reference frame $R''v$. In this vector sub-space, the parameters of space are inverted (minus sign). The laws of transformation in this reference frame are the product, for the concerned parameter, of the same affine transformation as in the passage of Rr in $R''r$ by a homothety of $-tg(\theta)$ ratio and of which center is the origin.

The «purely relativist» homothety (on the $\bar{x}''$ coordinate), bound to the invariance of the speed of light in the change of reference frame, is valid whatever is the speed of the «mobile» reference frame. Its ratio of homothety ($-tg(\theta)$), depends only on the instantaneous speed of the «mobile» reference frame with regard to the «fixed» reference frame.

Geometrical construction

Giving a vector of **λ** modulus (which represents an element of length **λ**) in the fixed referential frame. This vector is parallel to the axis of movement of the mobile referential frame. The image of this vector in the mobile referential frame is given by the geometrical construction of the figure 4:

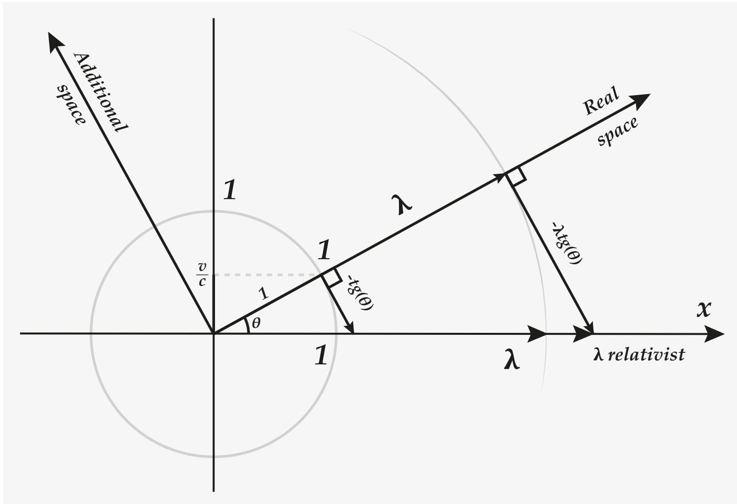

fig. 4 : Geometrical construction

Expression of time

We regard time as vector space of dimension 1. It is usually represented by a real scalar on a directed axis fitted with a norm. Let's suppose :

$\vec{t}$ = vector time,

t = real scalar,

$\vec{u}$ = unit vector time.

Let us associate with our reference frame of space (with 6 dimensions) a vector space of dimension 1 representing the time.

We will now proceed just as for the parameters of geometrical space by making an extension of the vector space representing the time (second axis of coordinates).

We use the same properties of the scalar product, applied to the system of axis "real time" and "supplementary" time (orthonormal axis) defining a vector space of dimension 2. The angle of 2 "time" vectors is defined in the same manner as previously, starting from the scalar product.

The second axis of "time" coordinates thus possesses a unit vector $\overset{\Rightarrow}{u}$.

We make the same rotation of angle θ in the space defined by vectors $\vec{u}$ and $\overset{\Rightarrow}{u}$ associated to the reference frame $R''c$.

Under these conditions, after application of the transformation equations of LORENTZ and having made the rotation of angle θ, such as defined above, the vector expression of time in wide $R''c$ is written :

$$\begin{cases} t'' = t - \frac{vx}{c^2} \\ \bar{t}'' = -\left(t - \frac{vx}{c^2}\right) tg(\theta) \end{cases}$$

We have an expression of t on the "real" axis which corresponds to an affine transform closely connected (respecting the law of addition of speeds).

On the "supplementary" axis :

The transform of the "time" coordinate takes a similar form to that we had obtained for the coordinate of geometrical space x'.

The "real" coordinate in the mobile reference frame (after affine transform) is transferred on the supplementary axis with a coefficient depending on the instantaneous speed of the reference frame ($-tg(\theta)$), and thus with a minus sign. In this subspace, the time varies in opposite sense of its sense of variation on the "real" axis.

This additional component indicates a distortion of time in the mobile reference frame related to the invariance of light speed.

It is just a corrective term. The obtained result representing a negative variation of time in the supplementary space is not inconsistent with the principle of EINSTEIN according to which two events, connected by a cause and effect link in a reference frame, must be connected by a similar bond in another reference frame, i.e. the change of reference frame must prohibit that in a reference frame the effect precedes the cause.

Indeed, the principle stated above applies to the whole space, i.e. before rotation with the vector sum of the two vector subspaces that we built and, in that case, the submission remains true (see the figure 4 in which the parameter λ represents an interval of time). On the other hand, the above submission does not apply to only one of the two subspaces.

Variations of time in the fixed reference frame and in the reference frame having done the rotation:

$$t'' = t - \frac{vx}{c^2}$$

$$\bar{t}'' = -\left(t - \frac{vx}{c^2}\right) tg(\theta)$$

We are in the same point of fixed abscissa x, at the two successive moments respectively t_1 and t_2, v being constant.

$$t_2'' - t_1'' = t_2 - t_1 \; ; \; \bar{t}_2'' - \bar{t}_1'' = -(t_2 - t_1)\, tg(\theta)$$

Relativist dynamic

A particle of mass m_0 at "rest" is animated by a constant speed $\vec{u}$ .We suppose it to move in space out of any gravitational field (restricted Relativity).

$$m = \frac{m_0}{\sqrt{1 - \frac{\|\vec{v}\|^2}{c^2}}}$$

Expression of the moving mass:

The expression of the energy (mass) is connected to the choice of the reference frame. It is thus not an intrinsic parameter but a parameter of space, which leads to envisage a rotation of θ angle similar to the one we had defined previously for time and geometrical space.

We so consider the energy as a vector space with two dimensions, in a reference frame connected to the mass in movement, this one being motionless in the moving reference frame.

Under these conditions, the vector representing the mass is written, before rotation:

$$m' = \begin{bmatrix} \dfrac{m_0}{\cos(\theta)} \\ 0 \end{bmatrix}$$

After rotation, we have :

$$m = \begin{bmatrix} m_0 \\ - m_0 \, tg(\theta) \end{bmatrix}$$

The “real” part of the mass appears invariant.

The “supplementary” part appears with a negative value, (which by no means corresponds to antimatter, with the usual signification in the laboratory).

Note :

For $tg(\theta) = 1 \;\Leftrightarrow\; \theta = \pi / 4$

$\Leftrightarrow \; sin(\theta) = \sqrt{2} / 2 \;\Leftrightarrow\; v = c / \sqrt{2}$

Ithe absolute value of the mass in the “supplementary” space is equal to the absolute value of the mass in “real” space.

Geometry of space :

In our representation, the space appears in the “mobile” reference frame, after rotation, in the form of a direct sum of two vector subspaces made up one by the same vector space as in the “fixed” reference frame, the other by the transform of a vector subspace of this same vector space by a homothety of center the origin and ratio $-tg(\theta)$.

The reference frame in movement thus contains the same space as the fixed reference frame plus a not empty one “symmetric” space, containing parameters of geometric space, time and energy, i.e. well possessing a

physical reality, and supported by linearly independent axis of those of “real” space and fixed reference frame. This shows that the structure of the space is more complex than the perception we have of it in the fixed reference frame, i.e. in the Euclidean reference frame.

Graphic construction of this representation of space

The mobile reference frame having made a rotation of angle θ, the parameters being supported by the axis of the "supplementary" space result from the corresponding axis by a homothety of ratio $-tg(\theta)$, i.e. negative. The geometric representation that we are going to give will thus be build by using a rotation of angle $-\theta$.

Let us imagine two observers, the first in the fixed reference frame, the second in the mobile reference frame after rotation.

According to the foregoing, the first observer considering a parameter λ (distance between two points on the x-axis, time interval between two events, expression of a mass), the second observer will perceive the same parameter λ on a first axis of coordinates plus its image by homothety of ratio $-tg(\theta)$ on a second orthogonal axis to the first one, which can be represented on the following diagram :

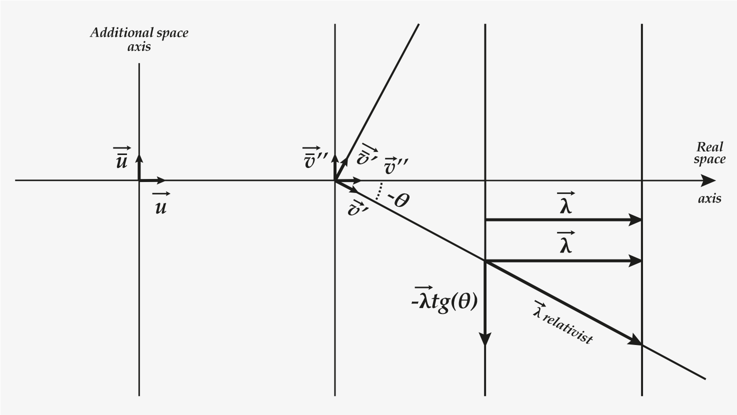

fig. 5 : Graphic construction of a relativist parameter

The equations of transformation of LORENTZ being valid whatever the speed, if the speed of the mobile reference frame varies on its axis of translation (axial acceleration), the angle of rotation of the mobile reference frame follows the variations speed of the mobile reference frame, which allows us to build an image of geometrical space in the case of an axial acceleration (see figure 6).

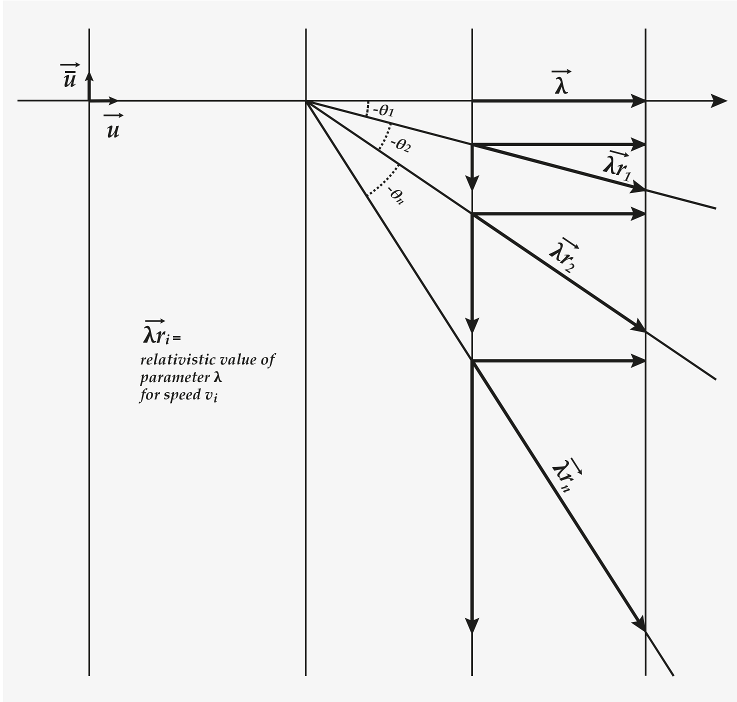

fig. 6 : Graphic construction of parameters with different speeds

Complete graphic construction :

We trace the graph (figure 7) of the function :

$y = -tg(arcsin(x))$ in an orthonormed reference frame of origin O.

We choose to represent the speed (= v) of the mobile reference frame with regard to the light speed :

(= v/c) by choosing $c = 1$.

In that case, x varies between 0 and 1. For a speed v (with $x = v$, $c = 1$), we draw in the point of abscissa x a parallel with the y axis.

It cuts the graph of y at the point $M (v, -tg(\theta)$.

From this point M, we draw the parallel line with the x-axis. It cuts the parallel line drawed with the y-axis from the point $(1,0)$ at the point P.

By construction, the straight line OP forms an equal angle to $-\theta$ with the x-axis.

After having carried forward the parameter **λ** on the ***x***-axis, we draw in extremities of the corresponding vector the parallels to the ordinate axis of our reference frame. The straight line ***OP*** intersects these 2 parallels with the ordinate axis by building, as previously, the vector range of **λ** by the equations of transformation of LORENTZ.

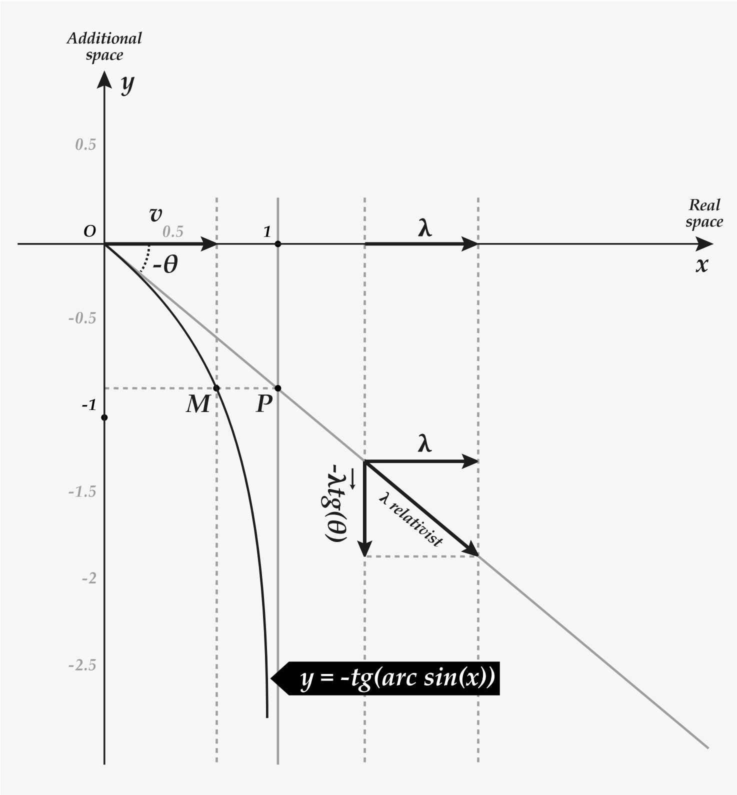

fig. 7 : Complete graphic construction of relativist parameters

This construction shows that speed is equivalent to an angle. This angle (the angle of rotation of the moving frame of reference) varies as the speed of the moving frame of reference is accelerated, resulting in a curvature of space.

Let's calculate the corresponding radius of curvature.

Curvature of space in the case of axial accelerated reference frames

In the fixed reference frame, the mobile reference frame has a movement of accelerated translation along the ***x*** axis of the fixed reference frame.

At the instant ***t***, the origin of the mobile reference frame is at the point ***O***. Given ***M*** any point of the axis of translation of the mobile reference frame (***x***-axis of the fixed reference frame). ***M′*** represents this point after rotation in the mobile reference frame. At this time, the origin of the mobile reference frame has a speed ***v*** and covers the distance $dx = v.dt$ during an infinitesimal interval of time ***dt***.

According to the foregoing, the image ***ds*** of this element ***dx*** in the mobile reference frame is represented (see figure 7) by a vector of coordinates:

$$ds = \begin{bmatrix} -v.dt \\ tg(\theta)\, v.dt \end{bmatrix}$$

At the point ***M′*** this differential element is on the right ***OP*** of which the gradient is $-tg(\theta)$ (according to the figure 7).

During the interval of time the gradient of the differential element varies of :

$$dp = -\frac{d\theta}{cos^2(\theta)} \quad ;$$

We suppose that the speed has a positive value ***v***.

It is always possible with a change of axis.

$$\sin(\theta) = \frac{v}{c} \Rightarrow \cos(\theta).\, d\theta = \frac{dv}{c} \Rightarrow d\theta = \frac{dv}{\sqrt{c^2 - v^2}}$$

We have thus :

$$dp = -\frac{dv}{\left(1 - \frac{v^2}{c^2}\right)\sqrt{c^2 - v^2}} \quad ; \quad dp = -\frac{c^2 .\, dv}{(c^2 - v^2)^{3/2}}$$

Thus, during the time interval dt the differential element ds rotates to an angle :

$$d\varphi = -\, arc\, tg\left(\frac{c^2 .\, dv}{(c^2 - v^2)^{3/2}}\right)$$

The radius of curvature R is defined by :

$$R = \frac{ds}{d\varphi} \quad \text{with} \quad ds = \frac{v .\, dt}{\sqrt{1 - \frac{v^2}{c^2}}} \quad ; \quad d\varphi < 0$$

For reason of the direction of the concavity of the curvature to the negative values of the x-axis of the reference frame.

Thus:

$$|R| = \frac{v .\, dt}{\sqrt{1 - \frac{v^2}{c^2}}\ arc\, tg\left(\frac{c^2 .\, dv}{(c^2 - v^2)^{3/2}}\right)}$$

$$|R| = \frac{c .\, v .\, dt}{\sqrt{c^2 - v^2}\ arc\, tg\left(\frac{c^2 .\, dv}{(c^2 - v^2)^{3/2}}\right)}$$

$$dt \to 0 \Rightarrow (ds \to 0 \ ; \ dv \to 0 \ ; \ d\varphi \to 0) \Rightarrow$$

$$|R| = lim_{dt \to 0} \frac{c .\, v .\, dt}{\sqrt{c^2 - v^2}\ \frac{c^2 .\, dv}{(c^2 - v^2)\ \sqrt{c^2 - v^2}}} \Rightarrow$$

$$|R| = c\, \frac{v}{\gamma}\left(1 - \frac{v^2}{c^2}\right)$$

with γ = instantaneous acceleration of the reference frame.

Remarks :

$\gamma = 0 \ \Rightarrow \ R = \infty$ (restricted Relativity – conditions of Lorentz)

At any time, the radius of curvature is the same at any point of the x-axis of the reference frame.

Center of curvature :

The center of curvature is over the perpendicular to the right OP at point M' bearing the element, oriented to the negative values of the x-axis of the referential frame.

M being a fixed point on the x-axis of the fixed reference frame, the set of the rights OP (O = accelerated moving origin of the mobile reference frame) constitutes the hull of the curve which represents the curvature of the space when t varies.

Complete representation of space in the case of accelerated reference frames

In order to describe not only space but also the movement in the change of reference frame, it is necessary to transform the laws of mechanics in the same way as the vector space was transformed. Since the laws of mechanics (differential equations) are linear forms and thus elements of the dual of the vector space, it is necessary to transform the dual in the same way as the vector space (same rotation and same homothety for the vector of the dual).

This, it is possible to conveniently construct a dual base from the vector space base. Then the transformation matrix is applied simultaneously to each corresponding vector of the dual base and to each corresponding vector of the dual base.

In the representation of the space we have constructed, the space is divided into two vector subspaces: the real subspace and the supplementary subspace.

The real sub-space is the same as the vector space represented in the fixed reference frame. It therefore has the same dual. The laws of the mechanics are therefore the same in these two space and vector subspace.

The supplementary subspace is obtained by the product a rotation and a homothety of a part of the real subspace. The same transformation must be applied to the dual. The rotation does not change the dual.

About the homothety:

After the rotation and before applying the homothety, the symmetric vector subspace is identical (isomorphic) to a part of the real subspace (corresponding to the translation axis). The dual of this symmetric subspace is thus the same as that of the corresponding part of the real subspace.

Thus, if we don't go any further, the same laws of mechanics would apply in the supplementary subspace and in the isomorphic part of the real subspace.

In fact, we apply simultaneously the homothety to the vectors of the supplementary subspace and to the elements of the dual i.e., concrete terms in our case, to the motion parameters (velocity and acceleration) that are linear applications (that belong to the dual) and that have been calculated in the corresponding part of the real subspace.

Finally, the image of the space in the supplementary subspace, including the motion, is obtained from the same homothety of a part of the real subspace and the corresponding motion in that part of the real subspace.

Thus, when the mobile reference frame is accelerated along an axis, in the real subspace of the mobile reference frame, which is the same as the vector space contained in the fixed reference frame, the expression of time t' in the mobile reference frame at the instant t of the fixed reference frame is represented by:

$$t' = t - \int_0^x \frac{v(x)\, d(x)}{c^2}$$

The abscissa x' of a point of the space at the instant t is represented by:

$$x' = x - \int_0^t v(t)\, dt$$

It is therefore possible to apply the laws of mechanics at a moment t, during a differential element of time, in the real subspace of the mobile reference frame, the laws of mechanics (defining the velocity and the acceleration) being the same as in the fixed reference frame.

It is then necessary to apply a homothetic relation to the abscissa of the corresponding point of the real space on the corresponding axis of the additional space, as well as a homothetic relation to its motion on the same axis.

The usual difficulties of representing space in accelerated Euclidean frames of reference result from the fact that the structure of space is more complex than the image we have of it in the Euclidean frame of reference.

We have therefore constructed an adaptation of Newtonian mechanics to the case of the invariance of the speed of light. This adaptation is impossible in the case of Einstein's Relativity because of the inadequacy of the geometry.

Important note

Einstein's principle of equivalence, according to which a reference frame subjected to a gravitational field is locally (at the place of this reference frame) equivalent to an accelerated reference frame, makes it possible to extend all that was known so far to the case of the presence of a gravitational field.

Thus, according to the method indicated here, it is possible to describe space and the motions in space in the presence of a gravitational field without the need to build a basis of the dual or to resort to the tensor calculus.

Approach of comprehension of the causes of the invariance of the speed of light

The Lorentz transformation equations are the mathematical translation of the phenomenon of the invariance of the speed of light. Removing this invariance invalidates the Lorentz transformation equations and thus the double structure of space, which is the geometric consequence. Similarly, the removal of the double structure of space - and hence the conservation of the Euclidean norm in the change of reference frame - invalidates the Lorentz transformation equations and hence the phenomenon of the invariance of the speed of light. The invariance of the speed of light is therefore directly related to this double structure of space.

The second vector subspace has no component in the fixed reference frame. It is therefore not perceived directly in the laboratory or in the telescope, but only indirectly through the breaking of the Euclidean norm in the moving Euclidean reference frame.

The 2nd vector subspaces are therefore distinct and necessarily nested. Geometric elements of zero dimension do not exist in physics (they only exist in mathematics). The space is thus pixellated, i.e. it remains in the form of pixels belonging to the 1st and pixels belonging to the 2nd vector subspace, in the manner of the white and black squares of the chessboard.

The light which is directly perceived, in the 1st vector subspace, remains contained and spreads in the 1st vector subspace (visible light). The light which can be in the 2nd vector subspace remains contained and spreads in the 2nd vector subspace (directly invisible subspace).

This movement is reminiscent of the movement of the bishop on the chessboard : there is the white bishop, who moves only on the white squares, and who never meets the black bishop, who moves only on the black squares. The invariance of the speed of light in space implies that the transfer time of a photon from a pixel of the vector subspace to an adjacent pixel belonging to the same vector subspace is a universal constant independent of the movement of the source.

FINAL NOTE

The attempt to highlight a negative gravitation can be considered in zones of low extension outside intense gravitational fields, in particular within not very dense gas clouds (therefore subject to low internal gravitational forces) far from any massive element.

This type of gas cloud has an internal temperature close to that of the surrounding environment (cosmic microwave background).

A negative gravitational force in such a gas would imply a dilatation of the gas (with a decrease in its temperature) and a rarefaction of the particles in its heart (shape of the cloud would be approximatively toric).

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This study is not based on any other recent or ongoing study. Only perfectly known elements of physics and mathematics. For any additional information, please refer to university courses in mathematics and physics at levels 1 and 2.

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« A problem without a solution is an il-posed problem »

This quote from Albert Einstein unintentionally illustrates the impossibility he himself encountered in representing the universe in its entirety : he reasoned in a 4-dimensional space (Minkowski space-time), whereas the universe can only be represented in a space of higher dimension.